

2.5 Measures of Position and Boxplots

Assignment:

Z - score is the number of standard deviations that a given score is above or below the mean.

The Z - scores enable us to compare scores from different distributions with different means and different standard deviations.

For a sample: $Z = \frac{x - \bar{x}}{s}$

For a population: $Z = \frac{x - \mu}{\sigma}$

Round answers to 2 decimal places. Z - scores with values between -2.00 and 2.00 are considered normal scores. Z - scores with values less than -2.00 and greater than 2.00 are unusual scores.

Ex #1. The mean of a set of data is 5.07 and its standard deviation is 3.39. Find the Z - score for a value of 9.65. Is it unusual?

Ex #2. Which is better : a score of 82 on a test with a mean of 70 and a standard deviation of 8, or a score of 82 on a test with a mean of 75 and a standard deviation of 4?

Fractiles are numbers that partition or divide an ordered data set into equal parts. There are percentiles, deciles, and quartiles.

For whole numbers where $1 \leq P \leq 99$, the P th percentile of a distribution is the value such that $P\%$ of the data fall at or below P . Percentiles divide the data into 100 equal parts. There are 99 percentiles. The 50th percentile is equal to the median of the data values. (P_1 , P_2 , P_3 , P_4 , ..., P_{99})

So the 85th percentile means that 85% of the data values are at or below this percentile.

Deciles divide the data set into ten equal parts. (D_1 , D_2 , D_3 , ..., D_9)

Quartiles divide the data set into four equal parts. (Q_1 , Q_2 , and Q_3)

The first quartile is also the 25th percentile.

The second quartile is also the 50th percentile or the 5th decile.

The third quartile is also the 75th percentile.

To compute a percentile of a score in a data set:

$$\frac{\text{count the number of scores less than } x}{\text{total number of scores}} \cdot 100$$

To compute Quartiles:

1. Rank the data from smallest to largest.
2. Find the median. This is Q2.
3. The first quartile (Q1) is the median of the lower half of the data. Do not include Q2 if odd number of scores.
4. The third quartile (Q3) is the median of the upper half of the data. Do not include Q2 if odd number of scores.

Summary: Q1 = median of the lower half of the data = P25
 Q2 = median of the data set = P50 or D5
 Q3 = median of the upper half of the data = P75

Interquartile range (IQR) = $Q_3 - Q_1$

1. It measures the spread of the middle half of the data.
2. It is not influenced by an outlier (extreme value).

Box and Whisker Plots or Boxplots

1. 5 - number summary of the data and its spread. The five numbers are the lowest value, Q1, Q2 (median), Q3 and the highest value.
2. To construct a boxplot
 - A) Draw a horizontal scale to include the lowest and highest data values. Show a break if you need to. Must use equal increments.
 - B) Draw a rectangular box from Q1 to Q3. Q1 and Q3 and the ends of the box.
 - C) Include a solid line through the box where Q2 is located.
 - D) Draw solid lines (called whiskers) from Q1 to where the lowest value is and from Q3 to where the highest value is.

Ex. Make a boxplot from the given data values:

2	5	7	8	8	11	12	14	20	23
23	25	26	27	28	29	31	36	36	42